DRAG COEFFICIENTS OF SPHERES FALLING IN DILUTE AQUEOUS SOLUTIONS OF LONG-CHAIN MACROMOLECULES

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by

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ABSTRACT

The drag coefficients of spheres falling dilute aqueous solu-
tions of long-chain macromolecules were investigated over a range of
Reynolds numbers based on the viscosity of water from 800 to 7 X 10^4.
Solutions used had polymer concentrations of 100 to 1000 weight parts
per million of three grades of Polyethylene Oxide, WSR-35, WSR-205
and WSR-N-3000, and concentrations of 10 to 1000 weight parts per
million of WSR-301. Reductions in drag were observed in only the
solutions of the two higher molecular weight polymers, and then only
at Reynolds' numbers greater than 10^4. For all concentrations of
WSR-301, the drag decreased with increasing Reynolds' numbers and for
a given Reynolds' number the drag was minimized at a concentration of
100 wppm.

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SYMBOLS AND ABBREVIATIONS

A   =   Maximum cross-sectional area: $\dfrac{\pi d^2}{4}$

B   =   Weight of fluid displaced by sphere

C_d =   Drag coefficient: $\dfrac{4}{3}\pi\left(\dfrac{\rho'}{\rho}-1\right)g\,\dfrac{d}{v^2} = \dfrac{f}{v^2}$

d   =   Sphere diameter in centimeters

f   =   Drag coefficient factor: $\dfrac{4}{3}\pi\left(\dfrac{\rho'}{\rho}-1\right)g\,d$

g   =   Gravitational constant: 980.16 cm/sec^2

k   =   Virtual mass coefficient

m   =   Mass of fluid displaced by sphere

P35   =   Polyethelene Oxide, WSR-35

P205   =   Polyethelene Oxide, WSR-205

P301   =   Polyethelene Oxide, WSR-301

PN3000   =   Polyethelene Oxide, WSR-N-3000

Re   =   Reynolds number based on viscosity of water: $\dfrac{\rho v d}{\mu}$

s   =   Vertical position (cm) from surface of fluid

$\dot{s}$ =   Velocity: ds/dt

$\ddot{s}$ =   Acceleration: d^2s/dt^2

v   =   Terminal velocity

W   =   Weight of sphere

wppm   =   Weight parts per million parts of water

α =   Ratio of velocity to terminal velocity: s/v

ρ' =   Density of sphere (grams per cubic centimeter)

ρ =   Density of water (1.00 grams per cubic centimeter)

λ =   Ratio of sphere diameter to cylinder diameter

μ =   Dynamic viscosity of water: 0.01 grams/sec-cm

1. Introduction

The earliest published work in the field of turbulent flow fric-
tion reduction in dilute polymer solutions appears to be that of B.A.
Toms (8), who studied solutions of polymethylmethacrylate in chloro-
benzene. Measuring rates of flow through pipes, he discovered that,
for a given pressure gradient, the rate of flow of the polymer solution
was greater than that of the pure solvent. Investigating dilute
aqueous solutions of Poly(ethelene oxide), in "micropipes", Merrill (4)
found that for lower molecular weights no drag reduction occurred
until the transition from laminar to turbulent flow was complete;
however, as the molecular weight of the polymer was increased, drag
reduction occurred in the region of laminar-turbulent transition.

As a result of experiments with a rotating-disc apparatus, Hoyt
(2) has reported a 40% reduction in driving torque in a concentration
of 10 weight parts of Polyox 301 per million parts of water. He also
showed that for a given concentration of polymer, as the speed of the
disc is increased, the percentage torque reduction increases.

The effect of similar solutions on the drag experienced by spheres
has also been investigated. In this type of flow, the largest compo-
nent of the drag force is turbulent wake drag, and Ruszczycky (5)
reports that for a constant sphere size, the drag is decreased as con-
centration is increased, reaching a minimum at 7500 wppm of P301, and
than increasing again as the solutions become more concentrated. Also,
at this concentration, and at two lower concentrations, 2500 and 5000
wppm, as the sphere size increases, the amount of drag reduction in-
creases. Since the terminal velocity of the spheres increases with
size, this result could also be interpreted as showing that for a con-
centration which is an effective drag reducer, the amount of drag

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reduction increases as the Reynolds number increases.

Investigations carried out by Prather and Hayes (1a) showed that for a large range of sphere diameters, all spheres fell slower in a solution of 5000 wppm of P301 that in pure water. This is in direct contradiction to the results of Ruszczycky. The purpose of the research reported here was to measure the drag experienced by spheres for a range of concentrations of various polymer solutions, in order to determine whether a sphere actually does experience a drag reduction. If there is a drag reduction, then the dependence of this reduction on concentration and Reynolds number would be determined.

In an unpublished report, received after the experiments of this thesis were completed, Lang and Patrick (3) report drag reductions in 200 and 1000 wppm concentrations of P301, only for Reynolds numbers greater than 1×10^4. The 200 wppm solution showed greater drag reduction from 1×10^4 to 1×10^5 Reynolds number, but the 1000 wppm solution became the better drag reducer at higher Reynolds numbers. For both concentrations the drag reduction increased with increasing Reynolds number.

The characteristics of the polymers used are listed in Table 1.1. These polymers are water-soluble and non-toxic, and are further characterized on extremely large lengths to diameter ratios, possibly as high as 165,000:1 for P301(2).

TABLE 1.1

Polymer Characteristics

Poly(ethelene oxide)	Molecular Wt.	Class
Polyox WSR-N-3000	Unknown	Low viscosity
Polyox WSR-35	0.2×10^6	High viscosity resin
Polyox WSR-205	0.6×10^6	" " "
Polyox WSR-301	4.0×10^6	" " "

The solutions were all mixed in distilled water, at room temperature, nominally 24°C, with the mixing time depending on concentration and mixer speed.

The nylon and steel spheres used in the tests were weighed on an electronic balance accurate to ± 0.0001 gram. The manufacturer's specifications claim an average deviation from sphericity of ± 0.001%. The specific parameters are listed in Table 1.2. The increase in density as the size of the steel spheres decreases is believed to be due to the effect of the chrome plating.

TABLE 1.2

SPHERE PARAMETERS

Sphere No. Steel	d Diam. (cm.)	ρ' g/cc	f $(cm/sec)^2$
0	2.545	7.79	22605.86
1	1.270	7.79	11256.03
2	1.111	7.79	9865.68
3	0.952	7.79	8444.24
4	0.793	7.80	7157.70
5	0.634	7.82	5655.28
6	0.475	7.84	4246.50
7	0.315	7.95	2857.05
8	0.236	7.97	2152.32
9	0.157	8.02	1441.26
Nylon			
SN	0.636	1.14	115.10
MN	0.952	1.14	174.11
LN	1.270	1.14	232.27

2.1 Falling sphere drag coefficient

For a falling sphere, two equations may be written for the force acting upon it. The first is the definition of the drag coefficient

$$Cd = \frac{DRAG\ FORCE}{\frac{1}{2}\rho A \dot{s}^2} = \frac{F}{\frac{1}{2}\rho \dot{s}^2 \frac{\pi d^2}{4}} \qquad (1)$$

where A is the maximum cross-sectional area, ρ is the density of the fluid and $\dot{s}$ is the velocity. If the sphere has reached terminal velocity, $\dot{s}$ equals v, and the drag force is equal to the unbalanced weight

$$F = \frac{1}{6}\pi d^3 (\rho' - \rho) g \qquad (2)$$

Solving for the drag coefficient gives

$$Cd = \frac{4}{3}\left(\frac{\rho'}{\rho} - 1\right) g \frac{d}{v^2} = \frac{f}{v^2} \qquad (3)$$

where ρ', d, and f are given in Table 1.2, g is the local gravitational constant, ρ the density of the fluid and v is to be measured. To determine the drag coefficient that characterizes a given sphere in a given fluid, the terminal velocity must be measured in an infinitely extended fluid, that is, the presence of finite boundaries must not effect the value of the drag coefficient.

We may also define a drag reduction ratio as the difference in drag coefficient in the test solution compared to water divided by the drag coefficient in water.

$$\text{Drag reduction ratio} = \frac{Cd_{H_2O} - Cd_{polyox}}{Cd_{H_2O}}$$
$$= 1 - \left(\frac{v_{H_2O}}{v_{polyox}}\right)^2 \qquad (4)$$

where the densities of the Polyox solution and water are assumed equal. This is true for the low concentrations used in this research.

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2.2 Wall and End Effects

In determining the effect of finite boundaries on the drag of a falling sphere, it is useful to define a ratio of sphere diameter to cylinder diameter, λ. Haberman (1) develops a wall correction factor for Newtonian fluids as a function of an infinite power series in λ which is derived from the integration of certain coefficients of the expansion of the Stokes' stream function for a fluid moving past a sphere. For the largest value of λ in these experiments, 0.15, his correction is 1.1. This would be divided into the measured drag co-efficient to give the "infinite fluid" drag coefficient. However, the usefulness of this correction is limited to the restricted range of Reynolds' numbers for which his expansion of the Stokes' stream function is valid, Re less than 2.0. Turian (9), using a less exact formula for wall effects in water, investigated the extension of the concept to non-Newtonian fluids and concluded that the wall effects are the same for both fluids.

In addition to the presence of the tank walls, the termination of the tank by a rigid bottom may also have an effect on the measurement of the terminal velocity. Turian also investigated this effect in both water and non-Newtonian fluids and concluded that the "end effect" is negligible for a fall distance greater than one cylinder radius above the bottom.

2.3 Fall Distance to Achieve Terminal Velocity

Another consequence of the finite fall-distance which cannot be neglected is the possibility that the larger spheres would require a greater fall distance to reach terminal velocity than was available in the drop tank used in the experiments. The equation of motion of an

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accelerating body, including a correction for the virtual mass (the mass of fluid carried along with the body) is (3)

$$\left(\frac{\rho'}{\rho} + K\right) m \, \ddot{s} = (W - B) - Cd \, \frac{1}{2} A \rho \, \dot{s}^2 \qquad (5)$$

where k is the virtual mass coefficient, m is the water mass displaced by the body, W is the weight of the body, B is the weight of fluid displaced by the body, $\ddot{s}$ is the acceleration, $\dot{s}$ is the velocity and other terms as defined previously. For steady-state conditions, the acceleration is zero and $\dot{s}$ equals the terminal velocity, v,

$$v = \left(\frac{W - B}{\frac{1}{2} A \rho \, Cd}\right)^{1/2}$$

For convenience, define a parameter $a = \dfrac{\left[(W - B)(\frac{1}{2} \rho A \, Cd)\right]^{1/2}}{(\rho'/\rho + K) \, m}$

and, with some rearrangement, equation (5) becomes

$$\ddot{s} = a v - \frac{\dot{s}^2}{v} \qquad (6)$$

Integrating, with initial velocity zero, gives

$$\dot{s} = v \, TANH(at) \qquad (7)$$

And, integrating again for the fall distance

$$S = \frac{v}{a} \, \ln\left[\cosh(at)\right] \qquad (8)$$

Letting the velocity, $\dot{s}$ equal 0.99v, the tanh(at) equals 0.99 and the ln[cosh(at)] is 1.96. The distance to reach 0.99 terminal velocity is given by

$$S(.99) = 1.96 \, \frac{v}{a} = \frac{1.96 \, m \left(\frac{\rho'}{\rho} + K\right)}{\frac{1}{2} \rho A \, Cd} \qquad (9)$$

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Substituting for m and A the values for the No. Ø sphere, and using

a turbulent flow approximation for the virtual mass coefficient,

k = 1.8 (3), gives

$$S_\emptyset (.99) = \frac{64}{C_d} \qquad \text{centimeters} \qquad (10)$$

for the distance for the largest sphere to reach 0.99 terminal velocity.

It is apparent that for any value of drag coefficient less than 0.64,

the distance required will be greater than 100 cm. A similar develop-

ment for the No. 1 sphere gives a value of 0.32 for the limiting drag

coefficient. Since the measured drag coefficients for these spheres

are smaller than these limits, an acceleration correction to the drag

coefficients must be used.

Setting the ratio of $\dot{s}$ to v equal to α, and solving equation (7)

for (at) gives

$$(at) = TANH^{-1} \alpha = \frac{1}{2} \ln \frac{1 + \alpha}{1 - \alpha} \qquad (11)$$

Solving equation (8), for the No. Ø sphere, gives

$$S(\alpha) = \frac{32.5}{C_d} \ln \left[COSH (at) \right] \qquad (12)$$

Setting the maximum available fall distance equal to 90 cm,

$$COSH (at) = e^{2.8 \, C_d} \qquad (13)$$

Using the exponential form of the definition of the cosh and equation

(11),

$$COSH (at) = \frac{1}{2} \left[e^{\ln \left(\frac{1 + \alpha}{1 - \alpha} \right)^{1/2}} + e^{\ln \left(\frac{1 + \alpha}{1 - \alpha} \right)^{1/2}} \right] \qquad (14)$$

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Finally, equating (13) and (14) to eliminate cosh(at) the relation between α and C_d is

$$e^{2.8\,C_d} = \frac{1}{2}\left[\left(\frac{1+\alpha}{1-\alpha}\right)^{1/2} + \left(\frac{1-\alpha}{1+\alpha}\right)^{1/2}\right] \qquad (15)$$

Squaring both sides, and solving for gives

$$\alpha^2 = \frac{e^{5.6\,C_d} - 1}{e^{5.6\,C_d}} \qquad (16)$$

By substituting a series of values for C_d into (16), we can then arrive at the equivalent values of α, the ratio of the velocity measured at 90 cm. to the terminal velocity. If the drag coefficient was calculated from the measured velocity

$$C_d\,(MEASURED) = \frac{f}{\dot{s}^2}$$

and $\dot{s}$ equals αv, then the corrected C_d will be

$$C_d\,(CORRECTED) = \frac{f}{\dot{s}^2}\,\alpha^2 = \alpha^2\,C_d\,(MEASURED)$$

Using (16) for various values of gives:

Cd (measured)	α^2	Cd (corrected)
0.30	0.81	0.243
0.35	0.86	0.301
0.40	0.89	0.356
0.45	0.92	0.414
0.50	0.94	0.470
0.55	0.95	0.522

A similar development for the No. 1 sphere gives a value of α^2 of 0.99 or greater for drag coefficients greater than 0.25, which is not significant compared to the experimental error.

The correction to be applied to the Reynolds' number is

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$$Re\,(\text{CORRECTED}) = \frac{Re\,(\text{MEASURED})}{\alpha}$$

3. Experimental Apparatus and Technique

3.1 <u>General</u>. The main item of equipment used in the experiments was a glass cylinder, one hundred and twenty centimeters high, with an inside diameter of sixteen centimeters. A smaller tank, 50 cm. high, diameter 8 cm., was used for the testing of P35. The cylinder was filled with the test solutions, and spheres of various specific gravity and sizes were dropped on the tank center line, from just below the surface of the solution. The falling spheres were illuminated from above by a strobotac, flashing at a known frequency, and photographed on 35mm film. Two meter sticks were attached to the sides of the tank to allow measurement of the vertical position of the falling sphere.

3.2 <u>The dropping mechanism</u>. In the early stages of the experiment, a magnetic system was used to hold the spheres, and the release of the sphere synchronized with the camera shutter opening. However, as the need arose to vary the specific gravity of the spheres, requiring the use of non-magnetic materials, a vacuum system was constructed, which was then used for all the spheres. This system consisted of a header assembly connected to a vacuum source through a tee-fitting and a water trap. One outlet led from the tee-fitting to a valve, which was used to make or break the vacuum. At the other end of the header assembly a threaded fitting accepted brass tubes of various diameters. The sphere to be dropped was placed at the end of an appropriately sized tube and was held there by the vacuum. Then the end of the tube was submerged in the solution until the sphere was completely immersed. When the valve was opened, the sphere would drop.

3.3 <u>Lighting and Photography</u>. A Chadwick-Helmuth Strobotac, running on visual setting, was positioned above the tank, illuminating the

entire solution. The intensity of the illumination was relatively constant along the length of the tank. The strobe was driven by a Hewlett-Packard 202CR Low Frequency Oscillator, and the frequency was measured to 0.1 cps by using a H-P 521C counter on a ten second count. The flash rate was chosen to provide a picture of the sphere approximately every centimeter, for the smaller spheres, and every three centimeters for sphere No. 3 and larger.

Photographs of the falling spheres were taken on Kodak Plus-X film (ASA 125) using a Beseler Topcon 35mm camera with a 58mm focal length auto-Topcor 1:1.8 lens. This gave a vertical field of view of 70cm at a distance of four feet. The aperture setting was f/2.0, and the exposure time was on bulb. With the variation in flash rate proportional to the velocity, and the exposure time determined by the time of fall, the number of flashes per exposure was constant enough to provide negatives of equal density and contrast for all sphere photographs. The film was developed in a 3:1 dilution of Kodak Microdol "X", to minimize graininess. The negatives were viewed in an ARL-Dietert spectrograph viewer, which gave an overall magnification of 0.9 actual size, with no distortion along the vertical axis. The meter sticks mounted alongside the tube were used to determine the position of the sphere.

3.4 Solution Mixing Procedures. Previous experiments have shown evidence that P301, and other high molecular weight polymers, are susceptible to mechanical degradation. High speed mixing appears to break the molecular chains (3).

In the course of these experiments, two mixers were used. The first, a high speed Fulton Labmotor, was used with a single-bladed

paddle; later, a variable-speed, constant-torque motor with a three-bladed paddle was employed.

The effect of mixing procedures can be shown by the change in terminal velocity in different solutions of the same concentration of polymer. The three solutions listed in Table 3.1, below, were tested at 100 wppm (P301). Solution A was mixed at a concentration of 300 wppm for four hours with the high speed Labmotor. Solution B was mixed at a concentration of 10,000 wppm for six hours with the same mixer. Solution C was mixed at a concentration of 1000 wppm for one and one-half hours with the slow speed mixer running at 200 rpm. All solutions were diluted to the same concentration, 100 wppm, using the same procedure.

TABLE 3.1

Terminal Velocity (cm/sec) Variation with Solution Preparation

Solution

Sphere No.	A	B	C	Water
1	184.2	163.8	206.44	162.4
3	157.3	154.2	169.91	141.8

Solution C, mixed for the shortest time at the lowest speed, is the most effective drag reducer; solution B mixed longer than solution C, and faster than A, is the least effective. Therefore, for maximum effectiveness, solutions of long-chain polymers should be mixed at a slow speed, and to shorten the mixing time, at the most dilute concentrations possible.

3.5 Data Reduction. Using the viewing procedure described above (3.3), the position of each sphere was recorded. The position of the sphere at each flash was then plotted against the flash number. This resulted in a curve which gradually approached a straight line as the velocity

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approached its terminal value. The terminal velocity was calculated
by multiplying the slope of the straight line portion of the curve
times the flash rate. With the exception of the No. Ø sphere, at
least five points were used to draw the line of constant slope. Posi-
tion data within one cylinder radius of the bottom were not used, to
minimize the "end effect". The drag coefficients were then calculated,
using equation (3), and the Reynolds' number based on water from the
definition

$$Re = \frac{v \, d \, \rho}{\mu}$$

where the value of μ, the viscosity of the solution, was taken to
be equal to the viscosity of the solvent, water. At the concentrations
used, viscometer measurements (2) have shown this to be a good approx-
imation.

Due to combination of high flash rate for the faster-moving spheres
and low flash rates for the slower ones, the experimental error in de-
termining the velocity remained relatively constant, and since this is
by far the most significant factor in the error in Reynolds' number and
drag coefficient, these errors, too, are constant over the range of the
experimental data. These errors are listed in Table 3.2. The error in
the concentration of the solutions was impossible to determine, but
careful mixing procedures and continuous washing down of all equipment
surfaces during the mixing process minimized any loss of additive.
Therefore the concentration error was assumed to be equal to the error
in the weighing of the amount of additive added.

Due to the effect of variations in mixing procedures, discussed
in Section 3.4, there will be a difference in a given concentration of

1000 wppm of the same polymer mixed on different days, but the dilution to lower concentrations was done in the same manner for all solutions, so the relative concentrations of a given polymer should be accurate.

TABLE 3.2

Experimental Errors

Parameter	Error
Velocity	3.5%
Reynolds' Number	5.2%
Drag Coefficient	8.0%
Concentration	0.5%

4.1 Preliminary Experiments

Before beginning the very time consuming task of preparing large quantities of Polyox solutions, a test of the experimental apparatus and procedures was made, by dropping sphere Nos. Ø, 1, 3 and 5 in distilled water. The purpose of this test was three-fold: First, to verify the predicted experimental errors. Second, to determine the need for wall and end corrections. And third, to provide velocity data for estimating the proper flash rates to be used in the experiment.

Each of the spheres was dropped five times, to provide some statistical data. The results are shown in Table 4.1. The No. Ø sphere did not attain terminal velocity and using a flash rate of 30 fps, there were not enough points accurately to determine a non-terminal velocity, which could be used for a corrected drag coefficient.

TABLE 4.1

Terminal velocity (cm/sec) for tests in distilled water

Drop Number

Sphere No.	1	2	3	4	5	Avg.
1	159.0	164.4	157.2	165.9	165.6	162.4
3	143.4	144.6	139.5	140.7	140.7	141.8
5	117.0	119.4	121.0	122.8	121.6	120.4

The drag coefficients calculated from the average, minimum and maximum velocities are presented in Figure 1. The agreement with the classical drag coefficient curve (5), is excellent for the No. 3 and 5 sphere; the larger error for the number 1 sphere is most probably due to the low flash rate used in this particular test. Since any error in the drag coefficient due to wall and end effects would result in a higher value of measured Cd, and the error here is on the low

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side, it seems safe to neglect any corrections of this type for sphere
No. 1 and smaller sizes.

The maximum error in the velocity measured in this test was $\pm$ 2.7%,
for the No. 1 sphere, and the minimum was $\pm$ 1.8% for the number 3
sphere. This is less than the predicted error of $\pm$ 3.5% (Table 3.2),
and the errors in Cd and Re are proportionately less also.

The next test was made to determine the effect of ageing on a
solution of P301, diluted to 100 wppm. Two drops were made, the first
immediately after mixing, the second 48 hours later. Since delays in
testing of this length were anticipated, later in the experiment, this
procedure would show whether these delays would significantly effect
the results. The drag coefficients calculated from this test are shown
in Figure 2.

The random variation in drag coefficient for the spheres with Re
greater than 1 X 10^4 indicates that the ageing process does not signif-
icantly change the drag reduction properties of the solution; the
values are within the predicted experimental error. Of greater sig-
nificance is the fact that at lower Reynolds' numbers, the drag is
increased. This effect became one of the primary areas of study for
later tests, as it began to appear that there was a critical Reynolds'
number, below which solutions of this type were no longer effective
drag reducers.

5. Experimental Results

The total number of valid data points for all concentrations is 177. Referring to the table of experimental errors (Table 3.2), a conservative definition of actual drag reduction would be a value of the drag reduction ratio greater than 0.1. Using this as the criterion, there are only 43 drop tests which showed a reduction in drag. There are no instances of drag reduction for any of the data taken in solutions of P35 and PN3000; together, these two solutions account for 53 of the total number of points. There were 28 data points taken in solutions of P205. Of these, only 9 showed a greater than 10% drag reduction, and seven of these nine occurred at Reynolds' numbers greater than 1×10^4. Of the remaining 96 points, all taken in P301, 34 show effective drag reduction, and 33 of these points occur at a Reynolds' number greater than 0.98×10^4.

Another method of analyzing the data is to consider the distribution of drag reduction relative to concentration. Looking again at the nine instances of drag reduction in P205, four occur in each of the 1000 and 500 wppm solutions, the other in 100 wppm. In the tests of P301, however, we find 15 of the 34 occuring in solutions of 100 wppm, 12 more, evenly distributed in 250, 75 and 50 wppm solutions, three in concentrations of 750 wppm, and two each in solutions of 1000 and 500 wppm.

Looking for the maximum drag reduction attained, we find that the P301 maximum is a reduction of 46%, at a Reynolds' number of 6.9×10^4, in concentrations of 100 and 250 wppm. The maximum value for the P205 solutions is 22% at a Reynolds' number of 5.96×10^4, in a 500 wppm solution.

The terminal velocity (with the exception of the Ø sphere, as noted), Reynolds' number and drag coefficient for each sphere in each concentration are presented in Tables 5.1 through 5.7, with the values of the drag coefficients for the Ø spheres corrected for acceleration.

Plots of drag coefficient versus water Reynolds number are presented in Figures 3 through 7, where some points have been omitted for clarity. The plot in Figure 3, based on tests of P35 in the small tank, is a very good example of the errors which may result from either neglecting the presence of wall effects or not obtaining terminal velocity. The drastic increase in drag coefficients as Reynolds number increases is not a real effect, but is most likely due to the limited fall distance (40-50 cm.), in which even the number 4 sphere has not yet attained terminal velocity. No attempt was made to correct these values for acceleration, since the procedure is an approximation at best, and the corrections would be extremely large for so short a fall distance. The remaining Figures (4-7) show again the dependence of the drag reduction effect on large Reynolds' number, and can be used to estimate the percentage drag reduction, optimum concentrations and general effectiveness of a particular molecule as a drag reducer.

Table 5.1   P35   (small tank)

Sphere	Concentration (wppm)		
	1000	500	100
1	- - -	140.8 1.79 567	142.0 (a.) 1.80 (b) 558 (c)
2	140.0 1.56 503	136.5 1.52 529	138.5 1.54 514
3	- - -	132.1 1.26 483	133.2 1.27 475
4	126.1 0.999 450	122.8 0.974 474	126.6 1.000 446
5	117.5 0.745 409	113.1 0.717 442	112.8 0.715 444
6	102.2 0.485 406	103.0 0.489 400	101.8 0.484 409
7	82.5 0.260 419	84.0 0.264 404	80.8 0.254 437
8	69.6 0.164 444	70.0 0.165 439	70.1 0.165 438
9	- - -	- - -	53.6 0.084 501

(a)   Velocity, cm/sec

(b)   Reynolds' number based on water, $\times 10^{-4}$

(c)   Drag coefficient, $\times 10^3$

Table 5.2   PN3000

Sphere	Concentration (wppm)		
	1000	500	100
$\emptyset 1$	- - -	213.6 5.59 430	212.8 5.42 470
1	157.3 1.99 455	163.0 2.07 423	155.1 1.97 467
2	149.5 1.66 441	150.0 1.66 438	145.5 1.62 466
3	143.2 1.35 411	137.9 1.31 443	140.5 1.34 427
4	129.5 1.03 426	128.5 1.02 433	130.1 1.03 423
5	117.0 0.742 413	117.5 0.745 409	121.2 0.789 384
6	102.2 0.486 406	106.7 0.507 373	103.3 0.491 397
7	86.2 0.272 384	84.4 0.266 400	86.2 0.272 384
8	72.1 0.170 414	71.9 0.170 416	72.3 0.171 412
9	51.3 0.081 547	50.3 0.079 569	56.2 0.088 456
LN	25.1 0.319 368	25.3 0.321 364	24.9 0.316 374
MN	20.9 0.199 399	21.0 0.199 396	21.6 0.206 373
SN	15.6 0.099 472	16.5 0.105 422	16.8 0.107 405

Table 5.3   P205

Sphere	Concentration (wppm)		
	1000	500	100
∅¹	- - -	233.6 5.96 364	. 219.0 5.77 433
1	182.2 2.31 338	180.5 2.29 345	157.8 2.00 452
2	170.8 1.90 337	165.6 1.84 360	150.5 1.67 435
3	- - -	- - -	138.5 1.32 440
4	143.3 1.14 348	137.4 1.09 379	132.5 1.05 407
5	126.1 0.800 373	124.3 0.788 366	119.9 0.760 393
6	- - -	110.8 0.526 346	106.9 0.508 371
7	- - -	87.9 0.277 370	88.1 0.277 368
8	73.2 0.173 401	68.5 0.162 458	- - -
LN	25.5 0.324 357	25.1 0.319 368	25.2 0.320 364
MN	21.0 0.200 395	21.4 0.204 380	21.8 0.208 366
SN	- - -	- - -	16.4 0.104 425

Table 5.4 P301 (single drops)

Sphere	Concentration				
	1000	750	500	250	100
1	165.6 2.10 410	174.3 2.21 370	171.9 2.18 380	164.3 2.09 416	163.8 2.08 419
2	153.3 1.70 420	160.2 1.78 384	160.5 1.78 383	– – –	– – –
3	144.6 1.37 404	149.2 1.42 379	146.1 1.39 396	152.2 1.44 364	154.2 1.47 355
4	130.8 1.o4 413	123.2 0.98 465	137.1 1.09 376	133.8 1.06 399	144.0 1.14 345
5	113.4 0.719 439	109.8 0.646 469	119.6 0.758 395	123.0 0.780 373	129.0 0.818 339
6	89.4 0.424 532	95.4 0.453 467	98.8 0.469 435	103.2 0.490 398	108.5 0.515 360
7	74.6 0.235 514	75.6 0.238 500	76.4 0.241 490	77.2 0.243 478	85.6 0.269 390
8	63.6 0.150 532	63.2 0.149 539	65.0 0.153 510	64.4 0.152 519	72.9 0.172 404
9	– – –	– – –	49.2 0.077 595	– – –	– – –

Table 5.5   P301 (100 wppm)

Sphere	1 hour [1]	48 hours [1]
1	184.2 2.34 333	185.1 2.35 328
2	170.4 1.89 340.	167.4 1.86 352
3	157.3 1.50 341	160.8 1.53 326
4	142.5 1.13 347	137.1 1.09 374
5	120.8 0.766 375	126.6 0.803 352
6	106.0 0.504 375	- - -
7	81.6 0.257 426	83.4 0.263 408
8	67.2 0.159 474	67.2 0.159 474
9	51.6 0.081 541	51.7 0.081 539

1. Elapsed time after mixing

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Table 5.6 P301   (Average values, 5 drops)

Sphere	Concentration (wppm)				
	1000	750	500	250	100
$\emptyset^1$	233.8 5.95 365	245.6 6.25 329	251.8 6.41 301	272.7 6.94 241	272.0 6.92 241
1	172.9 2.19 376	175.0 2.22 367	184.0 2.34 333	184.1 2.34 332	184.6 2.34 330
3	145.7 1.39 397	147.5 1.40 388	146.9 1.40 391	153.8 1.46 356	149.8 1.43 419
5	114.5 0.726 431	112.8 0.715 444	114.7 0.727 429	119.4 0.757 396	119.6 0.758 395

Table 5.7 P301   (Very dilute solutions)

Sphere	Concentration (wppm)				
	100	75	50	25	10
$\emptyset^1$	266.4 6.78 251	256.4 6.52 290	241.0 6.13 344	216.1 5.50 450	210.6 5.36 473
1	206.4 2.62 264	195.4 2.48 294	181.2 2.30 341	167.2 2.12 402	161.1 2.04 434
3	170.0 1.62 292	170.0 1.62 292	160.8 1.53 326	145.7 1.39 397	144.5 1.37 404
4	148.7 1.18 323	14914 1.18 320	144.7 1.15 342	135.2 1.07 391	138.0 1.09 375

1.   Drag coefficient corrected for acceleration

6. Conclusions

From the discussion of the results, it is obvious that P301 is the most effective drag reducer of the four polymers tested. If it is compared to the next best, P205, we see that a reduction of 22% (the maximum in a 500 wppm solution of P205), can be achieved in as little as 50 wppm of P301. The molecular weight of P205 is only 15% of that of P301, and the molecular structure is the same, that is, they have roughly the same weight per unit length. Therefore, it appears that the longer the molecule, the better it will be as a drag reducer. This has been verified by Hoyt for pipe flow, when he found that molecules heavier than Polyox were not as effective in reducing drag as Polyox if they had a smaller length to diameter ratio (2).

Examination of Figures 6 and 7 shows that for Reynolds numbers less than 10^5, the 100 wppm concentration of P301 exhibits the lowest values of drag coefficient. For more dilute solutions, the drag increases until the behavior is just like that of water for a concentration of 10 wppm. For more concentrated solutions, the drag increases and even goes above that of water at the lower Reynolds numbers when the concentration has reached 1000 wppm. At these higher concentrations the Reynolds number based on the viscosity of water is no longer meaningful (2) and the drag coefficient curves lose their usefulness.

Examining Figure 8, showing the drag reduction as a function of Reynolds number, it appears that a critical Reynolds number exists for this type of flow. This is similar to the results for the pipe flow experiments of Merrill. For the concentrations and polymers tested here, the critical Reynolds number is approximately 1×10^4. The variation in drag reduction near this critical value suggests that there

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may exist a region of unstable flow in polymer solutions at this Reynolds region, which sometimes flips into an effective drag reduction, and other times does not. Even in flow past a sphere in water, the transition from a laminar to a turbulent flow is an unclear region, with much dependence on surface roughness, sphere motion and intensity of turbulence in the water itself. Similar effects may influence the drag reduction effectiveness of polymer solutions.

Again, examining the relation between amount of drag reduction and Reynolds number, we see that as the Reynolds number increases, the amount of drag reduction for a given concentration increases also.

Briefly summarizing these experiments, we can say that for concentrations of long-chain macromolecules in aqueous solutions of 1000 wppm or less, in a range of water Reynolds numbers from 800 to 7×10^4:

1. Solutions in these concentrations may be effective drag reducers if their molecular weight is at least greater than 0.2×10^6, and their length to diameter ratio is very large.

2. The most effective drag reducer is P301. This effect is primarily due to its very large length to diameter ratio, approximately 6.7 times larger than P205.

3. Drag reduction occurs only for water Reynolds numbers greater than some critical number, approximately 10^4.

4. For Reynolds number less than 10^5, the maximum drag reduction was obtained at a concentration of 100 wppm of P301.

5. The amount of drag reduction increases with Reynolds number for a given concentration, provided that the solution is an effective drag reducer at some lower Reynolds number.

6. Below the critical Reynolds number, drag is increased or

remains the same as in water. The higher the molecular weight, the
more drag is increased in this region.

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ACKNOWLEDGEMENTS

FIG 1.

FIG. 2

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FIG. 3

DATA TAKEN USING SMALL TANK

P35

x : 1000 wppm
△ : 500
○ : 100
——— WATER

FIG. 4

FIG. 5

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Fig. 6

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FIG 7

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FIG.8

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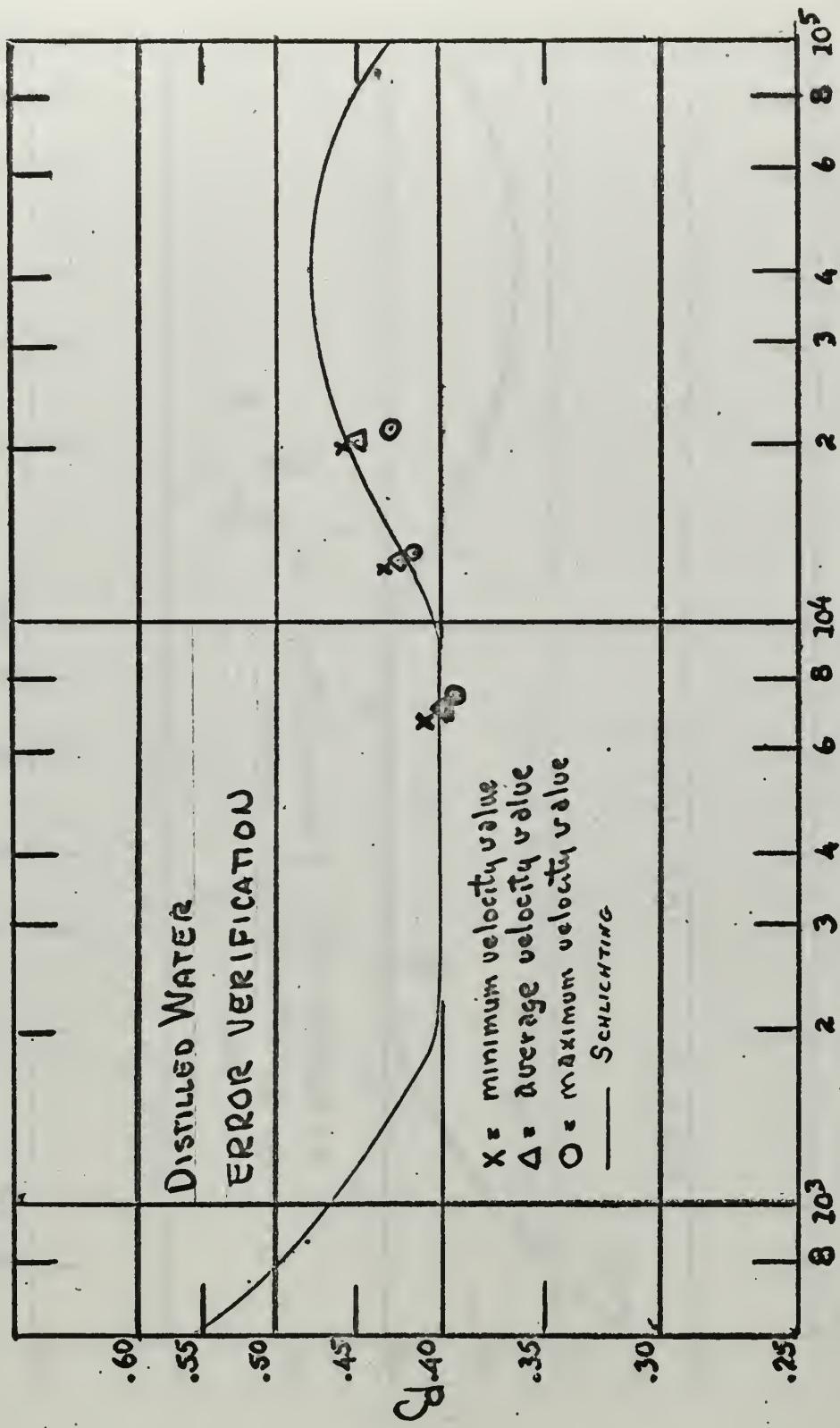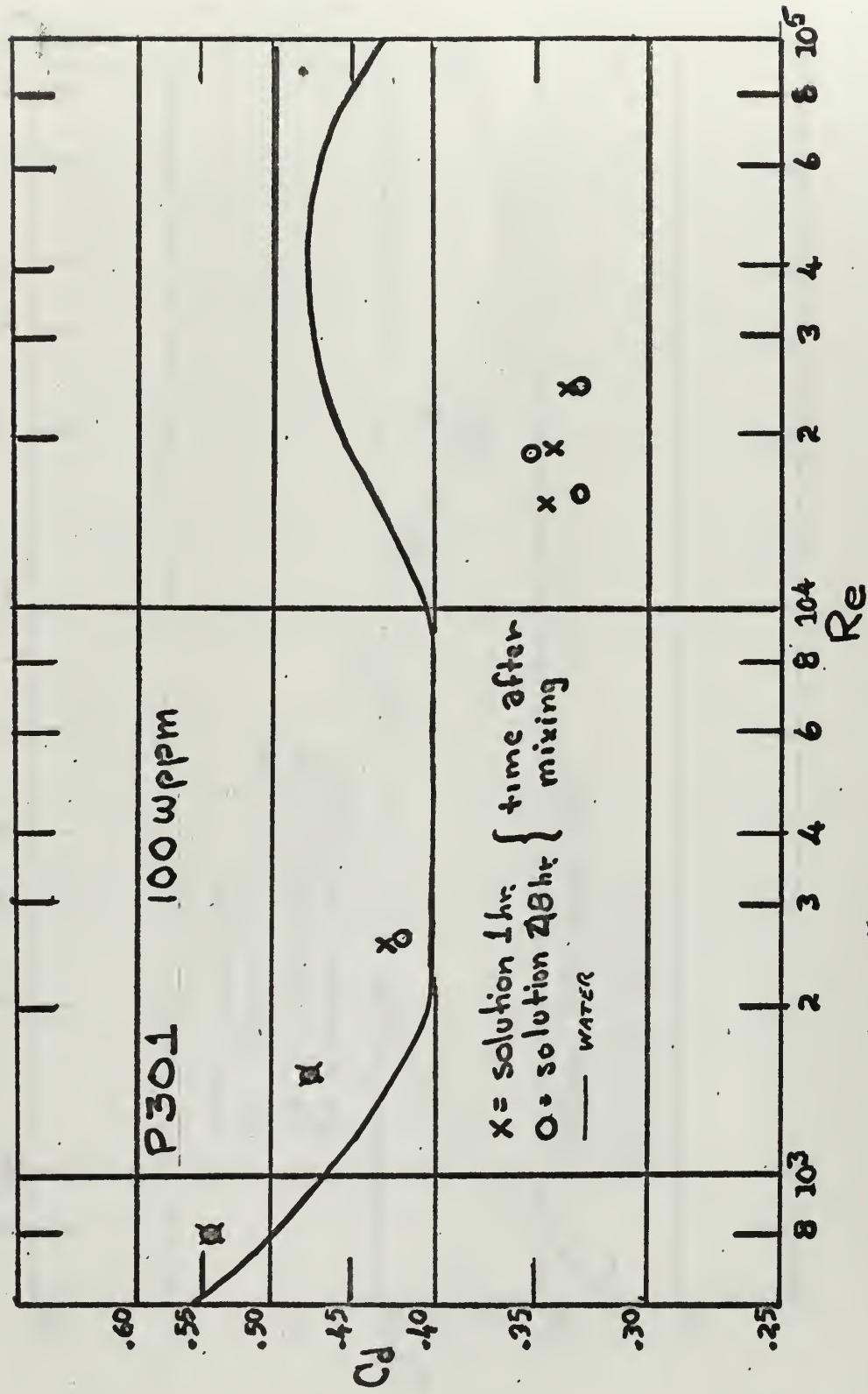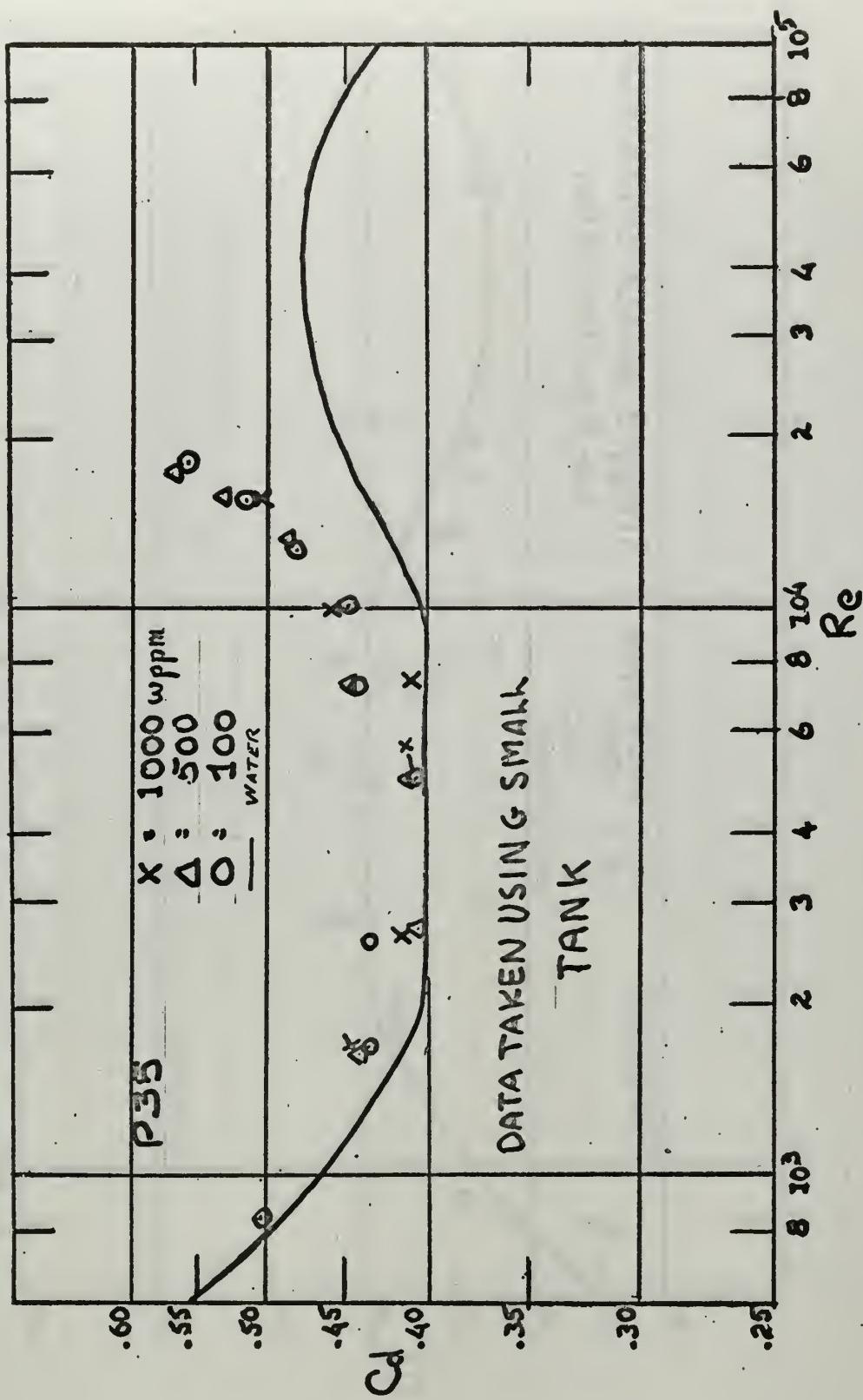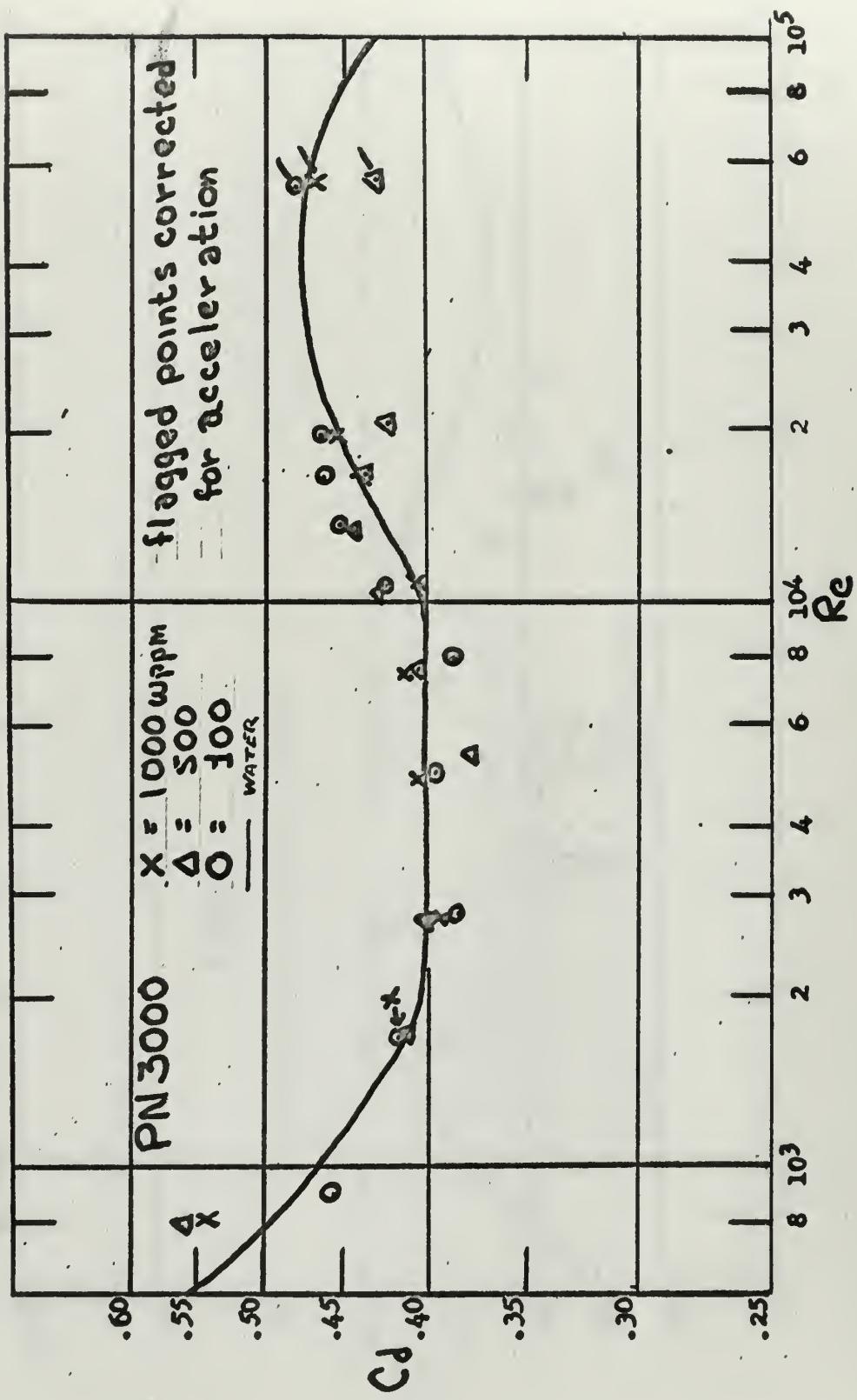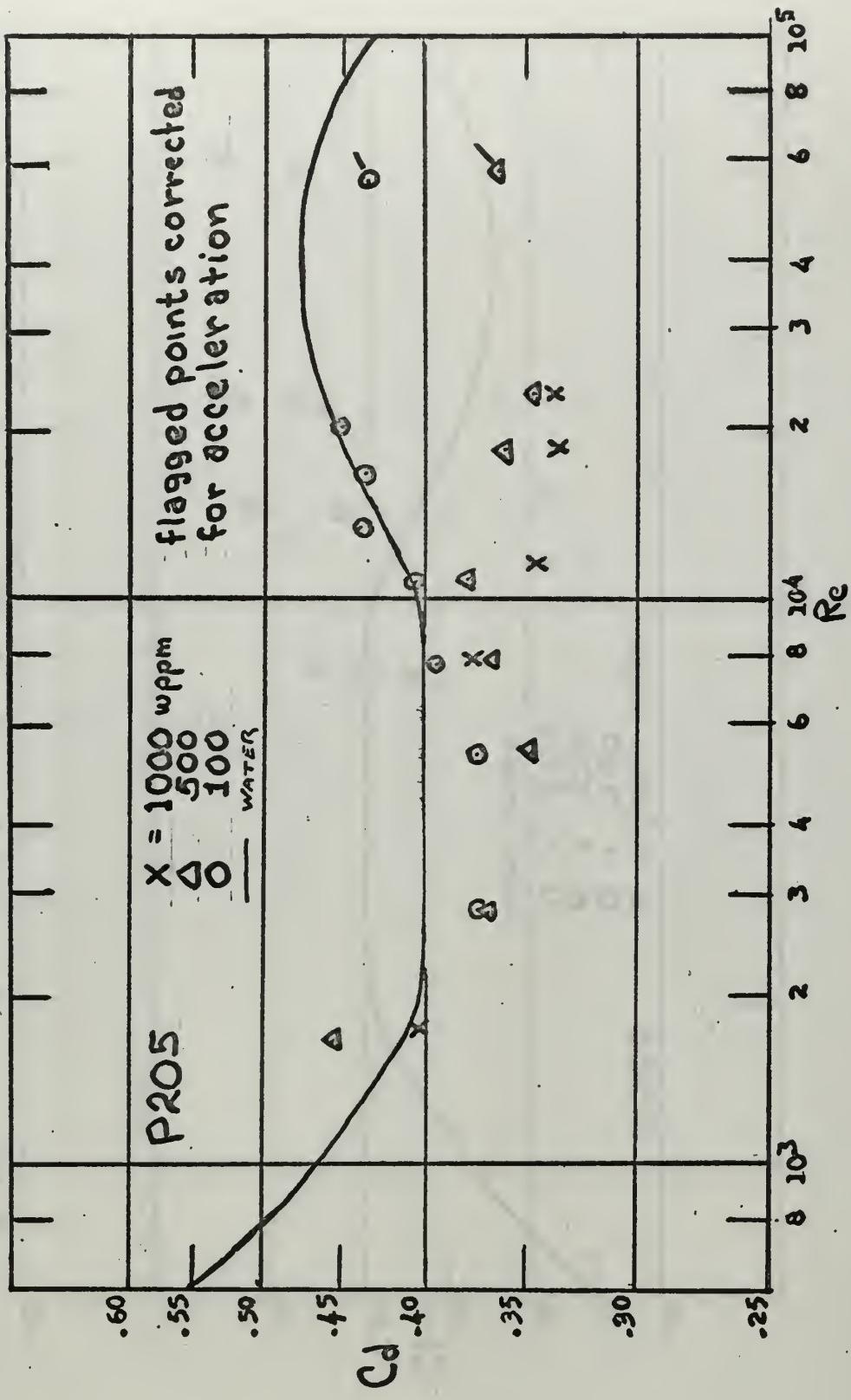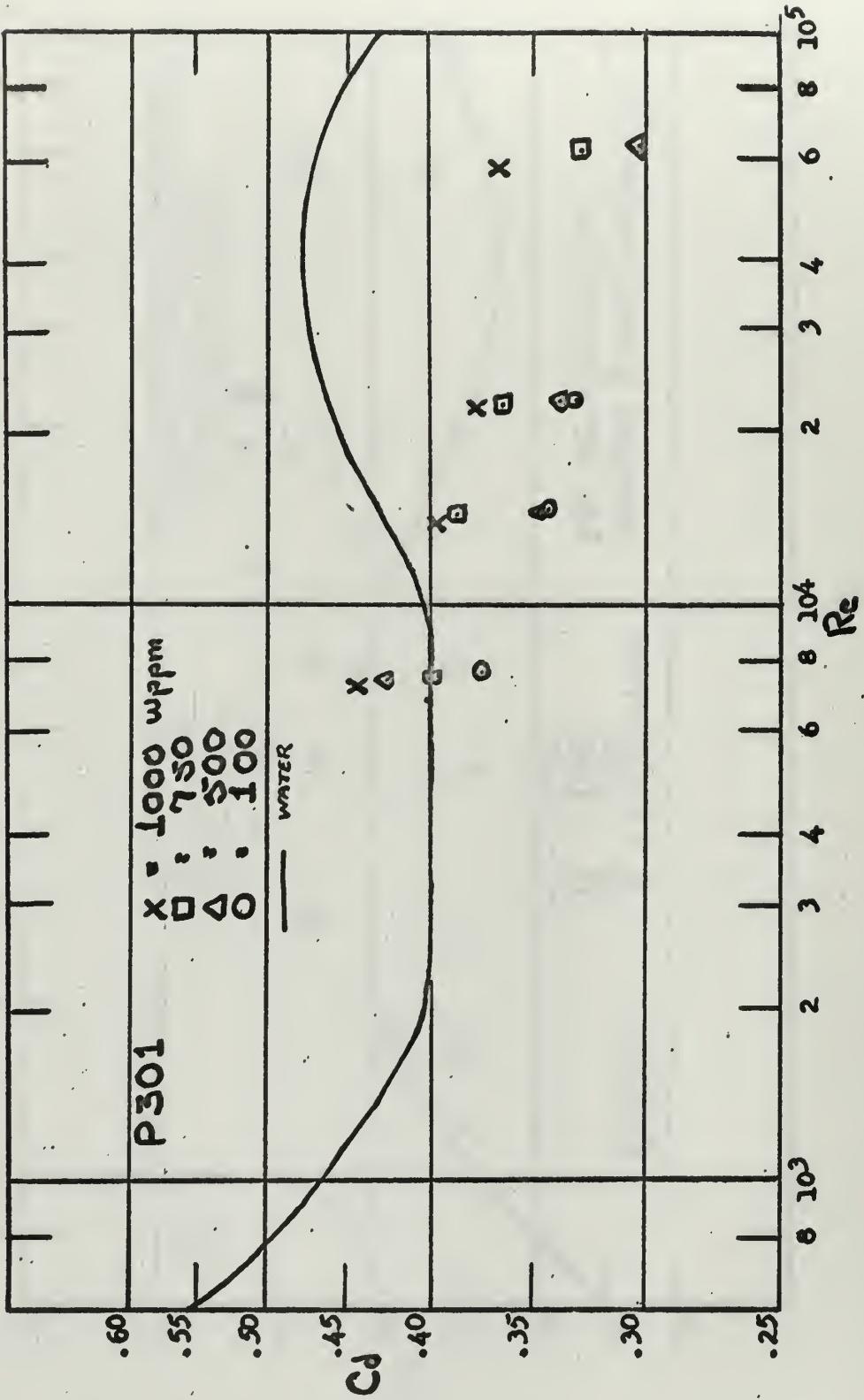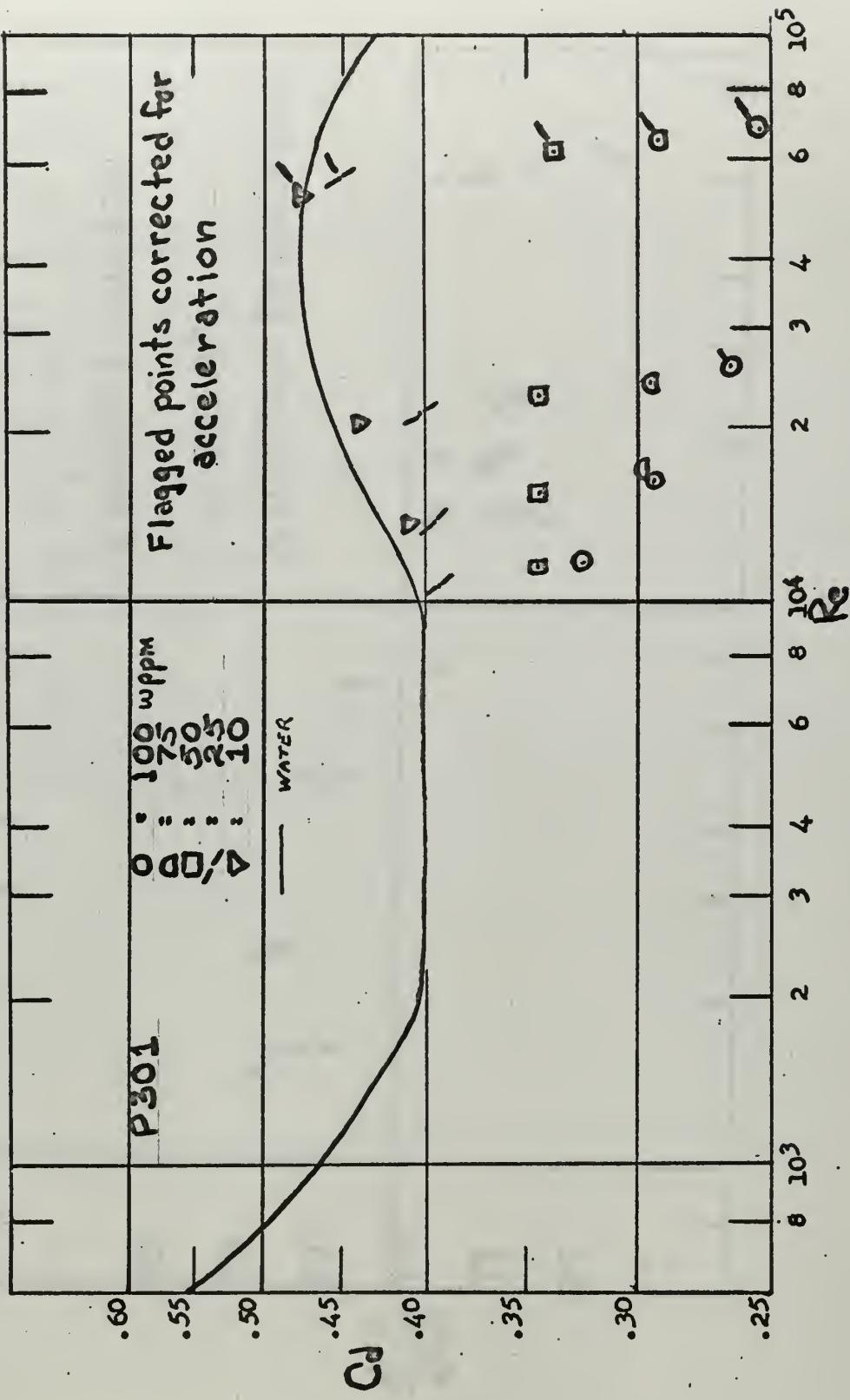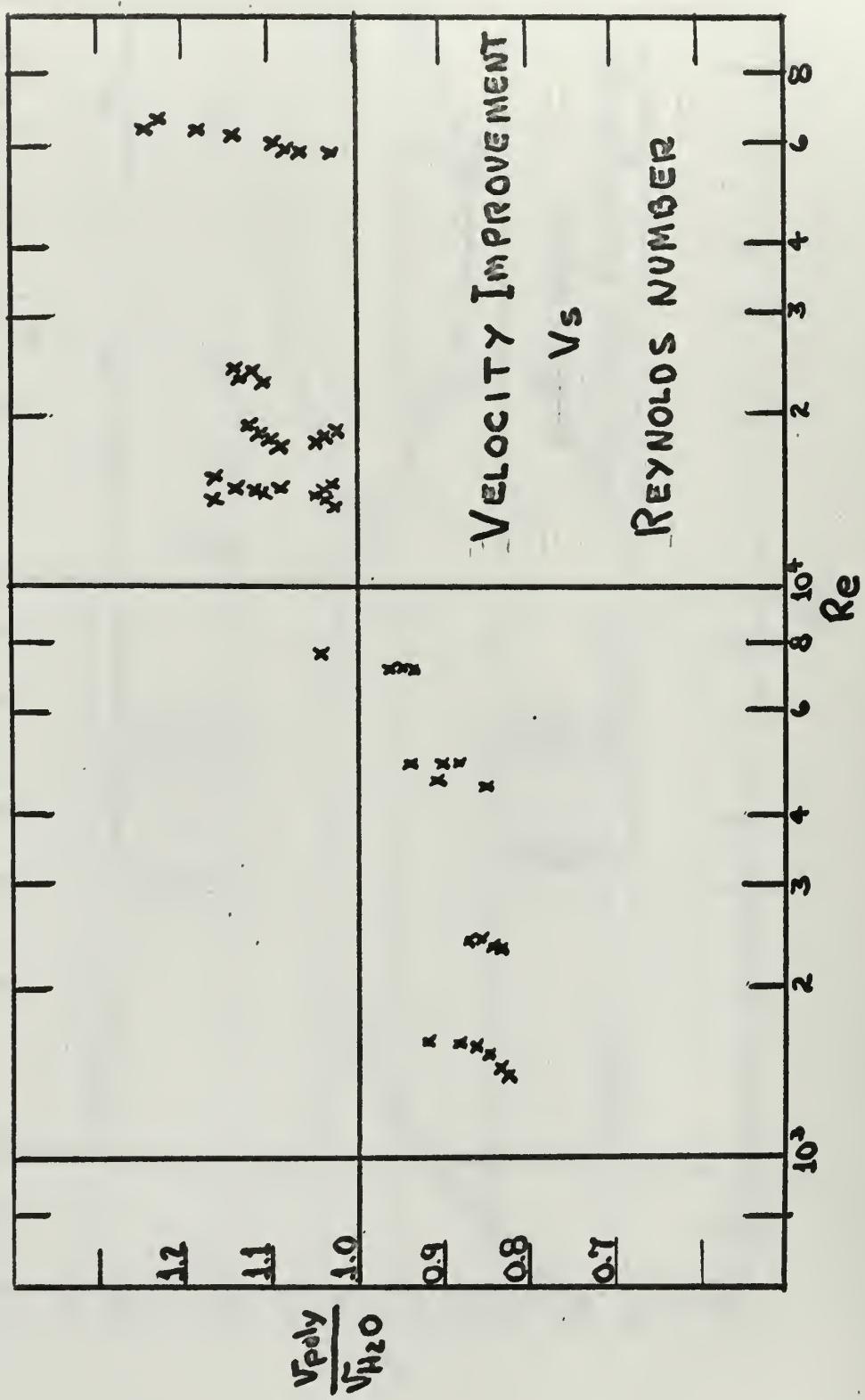
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The drag coefficients of spheres falling in dilute aqueous solutions of long-chain macromolecules were investigated over a range of Reynolds' numbers based on the viscosity of water from 800 to 7×10^4. Solutions used had polymer concentrations of 100 to 1000 weight parts per million of three grades of Polyethylene Oxide, WSR-35, WSR-205 and WSR-N-3000, and concentrations of 10 to 1000 weight parts per million of WSR-301. Reductions in drag were observed in only the solutions of the two higher molecular weight polymers, and then only at Reynolds' numbers greater than 10^4. For all concentrations of WSR-301, the drag decreased with increasing Reynolds' number and for a given Reynolds' number the drag was minimized at a concentration of 100 wppm. (U)

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	ROLE	WT	ROLE	WT	ROLE	WT
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